

$\Upsilon(1s) \rightarrow \gamma(\eta', \eta)$ Decays

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Abstract

The decays of $\Upsilon(1s) \rightarrow \gamma(\eta', \eta)$ are studied by an approach which has successfully predicted the ratio $\frac{\Gamma(J/\psi \rightarrow \gamma\eta')}{\Gamma(J/\psi \rightarrow \gamma\eta)}$. Strong dependence on quark mass has been found in the decays $(J/\psi, \Upsilon(1s)) \rightarrow \gamma(\eta', \eta)$. Very small decay rates of $\Upsilon(1s) \rightarrow \gamma(\eta', \eta)$ are predicted.

Recently, CLEO Collaboration has reported measurements[1] of the branching ratios of radiative decays of $\Upsilon(1s)$ into η and η'

$$\begin{aligned} B(\Upsilon(1s) \rightarrow \gamma\eta) &< 1.0 \times 10^{-6}, \\ B(\Upsilon(1s) \rightarrow \gamma\eta') &< 1.9 \times 10^{-6} \end{aligned} \quad (1)$$

at the 90%C.L. In an early search for $\Upsilon(1s) \rightarrow \gamma\eta'$ by CLEO[2] no signal in this mode has been found and in a 90% confident level the upper limit of the branching ratio is about 1.6×10^{-5} . The previous CLEO search of $\Upsilon(1s) \rightarrow \gamma\eta$ produced an upper limit of 2.1×10^{-5} at the 90% confidence level[3]. Comparing with $B(J/\psi \rightarrow \gamma\eta'(\eta))$, the branching ratios of $\Upsilon(1s) \rightarrow \gamma(\eta', \eta)$ are very small.

There are rich gluon physics in the radiative decays of heavy vector mesons to η' . In QCD these processes are described by $V \rightarrow \gamma gg$, $gg \rightarrow \eta'$. It is known for a long time that the coupling between η' meson and two gluons is strong. The U(1) anomaly[4] of η' meson is the first evidence. The quark components of η' meson contribute only about $\frac{1}{3}$ of $m_{\eta'}$, therefore, about $\frac{2}{3}$ of $m_{\eta'}$ comes from gluon components of η' [5]. Experimental data[6] show that $B(J/\psi \rightarrow \gamma\eta') > B(J/\psi \rightarrow \gamma\eta)$ and $BR(J/\psi \rightarrow \omega(\phi)\eta) > BR(J/\psi \rightarrow \omega(\phi)\eta')$. These results support that η' contains significant components of gluons[5]. Now very small upper limit of $B(\Upsilon \rightarrow \gamma\eta')$ has been reported by CLEO[1]. The branching ratio is smaller than $BR(J/\psi \rightarrow \gamma\eta') = (4.71 \pm 0.27) \times 10^{-3}$ by almost three order of magnitudes. This new

channel can be used to study the application of QCD to the radiative decays of Υ and the gluon content of η' .

The difference between the radiative decays of J/ψ and $\Upsilon(1s)$ is caused by the mass difference of c- and b-quark. As pointed out in Ref.[1] that the $B(\Upsilon(1s) \rightarrow \gamma\eta')$ predicted by the naive scaling is too large. There are different theoretical approaches in the study of the branching ratios of $\Upsilon(1s) \rightarrow \gamma(\eta', \eta)$. Very small branching ratios for $\Upsilon \rightarrow \gamma\eta, \gamma\eta'$ ($10^{-6} - 10^{-7}$) are obtained by employing the Vector Meson Dominance Model in Ref.[7]. The branching ratios of quarkonium decays to $\gamma\eta(\eta') \sim (5 - 10) \times 10^{-5}$, obtained in a nonrelativistic quark model[8] are larger than the upper limits(1). By using QCD sum rule and the U(1) anomaly, $B(\Upsilon \rightarrow \gamma\eta') = 3.3 \times 10^{-5}$ and $B(\Upsilon \rightarrow \gamma\eta) = 4.4 \times 10^{-6}$ are obtained in Ref.[9]. They are larger than the upper limits(1). It is pointed out by the author of Ref.[10] that using QCD sum rule and the U(1) anomaly[9], the decay width of $V \rightarrow \gamma\eta'$ has a factor of $\frac{1}{m_q^4}(q = c, b)$ for J/ψ and Υ respectively. This factor is the reason of larger $B(\Upsilon \rightarrow \gamma\eta')$ obtained in Ref.[9]. In Ref.[10] strong dependence on quark mass has been found. Using $m_\Upsilon \sim 2m_b$ and $m_{J/\psi} \sim 2m_c$, it is obtained

$$B(\Upsilon(1s) \rightarrow \gamma + \eta')/B(J/\psi \rightarrow \gamma + \eta') \sim 1.31(Q_b^2 m_c^6)/Q_c^2 m_b^6)(\alpha(m_c)/\alpha_s(m_b)),$$

$$B(\Upsilon(1s) \rightarrow \gamma\eta) \sim 3.3 \times 10^{-7}, \quad B(\Upsilon(1s) \rightarrow \gamma\eta') \sim 1.7 \times 10^{-6}.$$

They are consistent with the upper limits(1).

In 1984 we have studied $J/\psi \rightarrow \gamma\eta, \gamma\eta'$ [11]. In this short note we use the same approach to study $\Upsilon \rightarrow \gamma\eta'(\eta)$. A brief review of the study done in Ref.[11] is presented below. The decays are described as $J/\psi \rightarrow \gamma + g + g$ and two gluons are coupled to η, η' respectively. The decay amplitude of $J/\psi \rightarrow \gamma + g + g$ is calculated by pQCD. η and η' have gluon components

$$\langle G|T\{A_\alpha^a(x_1)A_\beta^b(x_2)\}|0\rangle = \frac{\delta_{ab}}{\sqrt{2E_G}}\epsilon_{\alpha\beta\mu\nu}(x_1-x_2)^\mu p^\nu f_G(x_1-x_2)e^{\frac{i}{2}p_G(x_1+x_2)}, \quad (2)$$

where G is a gluon state with quantum number of 0^{-+} . The function $f_G(x_1-x_2)$ is unknown. In Ref.[11] both a harmonic function with radius of one Fermi and $f_G(0)$ have been tried. The difference is about 10%. The possible dependence of $f_G(x_1-x_2)$ on x_1-x_2 has been ignored and f_G is taken as a parameter. The decay widths are derived as

$$\Gamma(J/\psi \rightarrow \gamma\eta') = \cos^2\theta \sin^2\phi \frac{2^{11}}{81} \alpha_s^2(m_c) \psi_J^2(0) f_G^2 \frac{1}{m_c^8} \frac{(1 - \frac{m_{\eta'}^2}{m_J^2})^3}{\{1 - 2\frac{m_{\eta'}^2}{m_J^2} + \frac{4m_c^2}{m_J^2}\}^2} \{2m_J^2 - 3m_{\eta'}^2(1 + \frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^2, \quad (3)$$

$$\Gamma(J/\psi \rightarrow \gamma\eta) = \sin^2\theta \sin^2\phi \frac{2^{11}}{81} \alpha_s^2(m_c) \psi_J^2(0) f_G^2 \frac{1}{m_c^8} \frac{(1 - \frac{m_\eta^2}{m_J^2})^3}{\{1 - 2\frac{m_\eta^2}{m_J^2} + \frac{4m_c^2}{m_J^2}\}^2} \{2m_J^2 - 3m_\eta^2(1 + \frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^2, \quad (4)$$

where θ is the mixing angle between η and η' and ϕ is the mixing angle between a 0^{-+} flavor singlet and a 0^{-+} glueball, $\alpha_s(m_c) = \frac{g^2}{4\pi}$ and g is the coupling constant of QCD, and $\psi_J(0)$ is

the wave function of J/ψ at the origin, which is determined by the decay rate of $J/\psi \rightarrow ee^+$

$$\psi_J^2(0) = \frac{27}{64\pi\alpha^2} m_J^2 \Gamma_{J/\psi \rightarrow ee^+}. \quad (5)$$

$m_c = 1.25 \pm 0.09 \text{ GeV}$ are presented in Ref.[6]. In the study of the amplitudes of $J/\psi \rightarrow \gamma + f(1273)$ [12] it is found that $m_c = 1.3 \text{ GeV}$ fits the data very well and this value is in the range of m_c [6]. Taking this value of m_c and $\theta = -11^\circ$, it is predicted[11]

$$\frac{\Gamma(J/\psi \rightarrow \gamma\eta')}{\Gamma(J/\psi \rightarrow \gamma\eta)} = 5.1, \quad (6)$$

which agrees with current experimental value $4.81(1 \pm 0.15)$ [6].

Eqs.(3,4) show strong dependence of the decay rates on m_c . This effect originated in the pQCD calculation of $J/\psi \rightarrow \gamma + g + g$ and the 0^{-+} gluon components of η , $\eta'(2)$. If $m_J = 2m_c$ is taken, the decay rates depend on $\frac{1}{m_c^2}$. However, the value $m_c = 1.25 \pm 0.09 \text{ GeV}$ [6] shows that $m_J = 2m_c$ is not a good approximation. Because of the cancellation between m_J and m_c in the factor of Eqs.(3,4)

$$\left\{ 2m_J^2 - 3m_{\eta'}^2 \left(1 + \frac{2m_c}{m_J} \right) - 16 \frac{m_c^3}{m_J} \right\}^2, \quad \left\{ 2m_J^2 - 3m_\eta^2 \left(1 + \frac{2m_c}{m_J} \right) - 16 \frac{m_c^3}{m_J} \right\}^2$$

the decay rate is very sensitive to the value of m_c . This sensitivity has been found in the study of the decay $J/\psi \rightarrow \gamma + f(1273)$ [12] too. In the right range of m_c [6] theory agrees with data very well.

By changing corresponding quantities and the electric charge of the quark, the decay rates of $\Upsilon(1s) \rightarrow \gamma\eta', \gamma\eta$ are obtained from Eqs.(3,4). The ratio is determined as

$$R_{\eta'} = \frac{B(\Upsilon \rightarrow \gamma\eta')}{B(J/\psi \rightarrow \gamma\eta')}$$

$$= \frac{1}{4} \frac{\alpha_s^2(m_b)}{\alpha_s^2(m_c)} \frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)} \frac{m_c^8}{m_b^8} \frac{(1 - \frac{m_{\eta'}^2}{m_\Upsilon^2})^3}{(1 - \frac{m_{\eta'}^2}{m_J^2})^2} \frac{(1 - 2\frac{m_{\eta'}^2}{m_J^2} + 4\frac{m_c^2}{m_J^2})^2}{(1 - 2\frac{m_{\eta'}^2}{m_\Upsilon^2} + 4\frac{m_b^2}{m_\Upsilon^2})^2} \frac{\{2m_\Upsilon^2 - 3m_{\eta'}^2(1 + \frac{2m_b}{m_\Upsilon}) - 16\frac{m_b^3}{m_\Upsilon}\}^2}{\{2m_J^2 - 3m_{\eta'}^2(1 + \frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^3} \frac{\Gamma_{J/\psi}}{\Gamma_\Upsilon}$$

where

$$\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)} = 4 \frac{\Gamma_{\Upsilon \rightarrow ee^+}}{\Gamma_{J/\psi \rightarrow ee^+}} \frac{m_\Upsilon^2}{m_J^2}. \quad (8)$$

Taking $m_J = 2m_c$ and $m_\Upsilon = 2m_b$, it is obtained

$$R_{\eta'} = \frac{B(\Upsilon \rightarrow \gamma\eta')}{B(J/\psi \rightarrow \gamma\eta')} = \frac{\Gamma(\Upsilon \rightarrow \gamma\eta')/\Gamma(\Upsilon \rightarrow \text{light hadrons})}{\Gamma(J/\psi \rightarrow \gamma\eta')/\Gamma(J/\psi \rightarrow \text{light hadrons})} \frac{B(\Upsilon \rightarrow \text{light hadrons})}{B(J/\psi \rightarrow \text{light hadrons})}$$

$$= \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \left(\frac{m_c}{m_b}\right)^7 \frac{1 - \frac{m_{\eta'}^2}{4m_b^2}}{1 - \frac{m_{\eta'}^2}{4m_c^2}} \frac{B(\Upsilon \rightarrow \text{light hadrons})}{B(J/\psi \rightarrow \text{light hadrons})} \frac{\Gamma_{\Upsilon \rightarrow ee}}{\Gamma_{J/\psi \rightarrow ee}} = 0.29 \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \left(\frac{m_c}{m_b}\right)^7. \quad (9)$$

Comparing with the ratio obtained in Ref.[10], stronger dependence on quark masses and small coefficient are obtained by this approach.

As mentioned above that $m_J = 2m_c$ is not a good approximation. In Ref.[6] $m_b = (4.7 \pm 0.07)$ GeV is listed. $m_\Upsilon = 2m_b$ works well. In this note $m_b = 4.7$ GeV and $m_c = 1.3$ GeV are taken. Because of strong dependence on quark mass both $\Gamma(J/\psi \rightarrow \gamma\eta', \gamma\eta)$ and $\Gamma(\Upsilon \rightarrow \gamma\eta', \gamma\eta)$ are sensitive to the values of m_c and m_b respectively. Inputting $B(J/\psi \rightarrow \gamma\eta')$, from Eq.(7) it is obtained

$$B(\Upsilon \rightarrow \gamma\eta') = R_{\eta'} B(J/\psi \rightarrow \gamma\eta') = 1.04 \times 10^{-7}, \quad (10)$$

$$B(\Upsilon \rightarrow \gamma\eta) = 0.022B(\Upsilon \rightarrow \gamma\eta') = 0.23 \times 10^{-8}. \quad (11)$$

Both branching ratios are less than the experimental upper limit[1].

The approach used in Ref.[11] is extended to study the decays of $\Upsilon(1s) \rightarrow \gamma\eta'(\eta)$. Very strong dependence of the decay rate on quark mass is revealed. The ratio $\frac{\Gamma(J/\psi \rightarrow \gamma\eta')}{\Gamma(J/\psi \rightarrow \gamma\eta)}$ and very small branching ratios of $\Upsilon(1s) \rightarrow \gamma\eta'(\eta)$ are predicted. The predictions agree with the data very well.

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